



# PHASE 3 MATHS WORKSHOP

ADDITION, SUBTRACTION,  
MULTIPLICATION AND DIVISION

Miss D.Hay



## AIMS:

- To review the stages of the calculation policy taught in phase 3.
- To explain how the children are expected to apply the skills to problem solving activities.
- To provide parents/carers with the information that they need to support their children at home.



# What is taught in school?

Primary mathematics is divided into four areas:

- **Using and applying mathematics** (problem solving, reasoning and communicating)
- **Number** (addition, subtraction, multiplication, division, decimals, fractions, percentages, algebra, estimation)
- **Shape, space and measures** (properties of flat and solid shapes, standard units of time, length, weight, capacity)
- **Data handling** (recording and interpreting information using lists, tables, graphs, diagrams)



# ADDITION



Progression across the year groups		
Addition		
	Typical calculations	Suitable methods
Y1	U+U TU + U (to 20 including zero)	Practical Number line
Y2	TU + U TU + multiples of 10 TU + TU U + U + U	Practical Number line Expanded columnar
Y3	HTU + U HTU + TU HTU + HTU	Number line Expanded columnar Column
Y4	THTU + HTU THTU + THTU	Expanded columnar Column
Y5	THTU.t + THTU.t THTU.th + THTU.th	Expanded columnar Column
Y6	THTU.tht + THTU.tht	Column



### Stage 3: Partitioning (expanded columnar method)

$$48 + 36 = 84$$

	40	8	
+	30	6	
	80	4	84

10

$$148 + 36 = 184$$

	100	40	8	
+		30	6	
	100	80	4	184

10

Carry

This builds on children's mental maths skills of partitioning and recombining  
 $40 + 30 = 70$   
 $8 + 6 = 14$   
 $48 + 36 = 84$



## Stage 4: Column (efficient)

$$\begin{array}{r} 48 \\ + 36 \\ \hline 84 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 148 \\ + 36 \\ \hline 184 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 48.56 \\ + 32.23 \\ \hline 80.79 \\ \hline 1 \end{array}$$

Children should be encouraged to estimate their answers first



# SUBTRACTION



Progression across the year groups		
Subtraction		
	Typical calculations	Suitable methods
Y1	U-U TU - U (to 20 including zero)	Practical Number line
Y2	TU - U TU - multiples of 10 TU - TU U - U - U	Practical Number line Expanded columnar
Y3	HTU - U HTU - TU HTU - HTU	Number line Expanded columnar Column
Y4	THTU - HTU THTU - THTU	Expanded columnar Column
Y5	THTU.t - THTU.t THTU.th - THTU.th	Expanded columnar Column
Y6	THTU.tht - THTU.tht	Column



### Stage 3: Partitioning (expanded columnar method)

	<sup>60</sup> <del>70</del>	<sup>1</sup> 4	
-	20	7	
	40	7	47

	100	<sup>60</sup> <del>70</del>	<sup>1</sup> 4
-		20	7
	100	40	7

Exchange  
Regroup



## Stage 4: Column (efficient)

$$\begin{array}{r} 61 \\ 74 \\ - 27 \\ \hline 47 \end{array} \qquad \begin{array}{r} 61 \\ 174 \\ - 27 \\ \hline 147 \end{array} \qquad \begin{array}{r} 48.56 \\ - 32.23 \\ \hline 16.33 \end{array}$$

Children should be encouraged to estimate their answers first



# MULTIPLICATION



Progression across the year groups		
Multiplication		
	Typical calculations :	Suitable methods
Y1	U x U	<b>Practical</b> (repeated addition) <b>Practical and pictorial arrays</b>
Y2	U x U	<b>Practical</b> (repeated addition) <b>Practical and pictorial arrays</b>
Y3	TU x U	<b>Grouping</b> on a number line progressing into <b>Expanded (grid)</b> and into <b>Short</b>
Y4	TU x U HTU x U	<b>Expanded (grid)</b> progressing into <b>Short</b>
Y5	HTU x U THTU x U  TU x TU	<b>Expanded (grid)</b> progressing into <b>Short</b>  <b>Expanded (grid)</b> progressing into <b>Long</b>
Y6	THTU x U  TU x TU  HTU x TU THTU x TU  U.i x U U.th x U  U.i x TU U.i x TU	<b>Short</b>  <b>Expanded (grid)</b> progressing into <b>Long</b>  <b>Long</b>  <b>Expanded (grid)</b> progressing into <b>Short</b>  <b>Expanded (grid)</b> progressing into <b>Long</b>



### Stage 3: Partitioning (grid method)

$$24 \times 3 = 72$$

X	20	4	
3	60	12	72

$$24 \times 32 = 768$$

X	20	4	
30	600	120	720
2	40	8	48
			768



## Grid method and decimals

<b>x</b>	<b>7</b>	<b>• 0.4</b>	<b>0.02</b>
7	49	2.8	0.14

$$\begin{array}{r} 49.0 \\ + 2.94 \\ \hline 51.94 \end{array}$$



## Stage 4: Short (column)

$$24 \times 3 = 72$$

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline 1 \end{array}$$

$$241 \times 3 = 723$$

$$\begin{array}{r} 241 \\ \times 3 \\ \hline 723 \\ \hline 1 \end{array}$$

$$1241 \times 3 = 3723$$

$$\begin{array}{r} 1241 \\ \times 3 \\ \hline 3723 \\ \hline 1 \end{array}$$



## Stage 5: Long (column)

$$24 \times 32 = 768$$

$$\begin{array}{r} 24 \\ \times 32 \\ \hline 48 \\ 720 \\ \hline 768 \end{array}$$

$$1245 \times 13$$

$$\begin{array}{r} 1245 \\ \times 13 \\ \hline 3735 \\ 12450 \\ \hline 16185 \end{array}$$

In the examples given, it is also correct to multiply starting with the tens digit (ie multiplying by the most significant digit first)



## Stage 5: Long (column)

$$24 \times 32 = 768$$

$$\begin{array}{r} 24 \\ \times 32 \\ \hline 48 \\ 720 \\ \hline 768 \end{array}$$



$$\begin{array}{r} 24 \\ \times 32 \\ \hline 8 \\ 40 \\ 120 \\ 600 \\ \hline 768 \end{array}$$

(4 x 2)  
(20 x 2)  
(4 x 30)  
(20 x 30)

$$24 \times 32 = 768$$

x	20	4	
30	600	120	720
2	40	8	48
			768



$$\begin{array}{r} 24 \\ \times 32 \\ \hline 600 \\ 120 \\ 40 \\ 8 \\ \hline 768 \end{array}$$

(20 x 30)  
(4 x 30)  
(20 x 2)  
(4 x 2)





# DIVISION



Progression across the year groups		
Division		
	Typical calculations	Suitable methods
Y1	U ÷ U TU ÷ U	Practical sharing Number-line grouping
Y2	U ÷ U TU ÷ U	Practical sharing Number-line grouping
Y3	TU ÷ U	Grouping on a number line progressing into Short
Y4	TU ÷ U  HTU ÷ U	Grouping on a number line progressing into Short  Short (remainders to be expressed as r)
Y5	HTU ÷ U THTU ÷ U	Short (remainders to be expressed as r, then as a fraction and as a decimal)
Y6	THTU ÷ U  HTU ÷ TU THTU ÷ TU  U.th ÷ U TU.th ÷ U HTU.th ÷ U THTU.th ÷ U	Short (remainders to be expressed as r, then as a fraction and as a decimal)  Long (remainders to be expressed as r, then as a fraction and as a decimal)  Short (remainders to be expressed as a decimal)



### Stage 3: Short division

$$372 \div 3 = 124$$

$$\begin{array}{r} 124 \\ 3 \overline{) 372} \end{array}$$

$$432 \div 15 = 28 \text{ r}12$$

$$\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{) 432} \end{array}$$

$$\begin{array}{r} 28 \frac{12}{15} \\ 15 \overline{) 432} \end{array}$$

$$\begin{array}{r} 28 \frac{4}{5} \\ 15 \overline{) 432} \end{array}$$

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432} \end{array}$$

remainder as a fraction

remainder as a decimal



## Short Division

$$432 \div 15 = 28 \text{ r}12$$

### Table of facts

$15 \times 1 = 15$   
 $15 \times 2 = 30$   
 $15 \times 3 = 45$   
 $15 \times 5 = 75$   
 $15 \times 10 = 150$

$$\begin{array}{r} 28 \text{ r}12 \\ \hline 15 \overline{) 432} \\ \underline{30} \phantom{0} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

### Jottings

$$30 = 15 \times 2$$

$$75 = 15 \times 5$$

$$\underline{45} = 15 \times \underline{3}$$

$$\underline{120} \qquad \qquad \underline{8}$$



Short Division

$$\begin{array}{r} 28 \text{ r} 12 \\ 15 \overline{) 432} \end{array}$$

Long Division

$$\begin{array}{r} 28 \text{ r} 12 \\ 15 \overline{) 432} \\ \underline{30} \phantom{2} \\ 132 \\ \underline{120} \\ 12 \end{array}$$



## Stage 4: Long division

$$560 \div 24 = 23 \text{ r}8$$

$$\begin{array}{r} 23 \text{ r}8 \\ 24 \overline{) 560} \\ \underline{48} \phantom{0} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

$$432 \div 15 = 28 \text{ r}12$$

$$\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{) 432} \\ \underline{30} \phantom{0} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \phantom{0} \phantom{0} \\ 132 \phantom{0} \\ \underline{120} \phantom{0} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

( $12 \div 15 = 0.8$ )  
remainder as a decimal

$$\begin{array}{r} 28 \frac{4}{5} \\ 15 \overline{) 432} \\ \underline{30} \phantom{0} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

( $0.8 = \frac{4}{5}$ )  
remainder as a fraction

With long division, there is the opportunity to teach an expanded method first (ie chunking)



## Long Division

$$432 \div 15$$

### Table of Facts

$$15 \times 1 = 15$$

$$15 \times 2 = 30$$

$$15 \times 3 = 45$$

$$15 \times 10 = 150$$

$$15 \times 5 = 75$$

$$15 \times 20 = 300$$

$$\begin{array}{r} \phantom{0}28r12 \\ 15 \overline{) 432} \\ \underline{300} \phantom{0} \\ 132 \\ \underline{120} \\ 12 \end{array}$$



## Long Division

### Table of Facts

$$24 \times 1 = 24$$

$$24 \times 2 = 48$$

$$24 \times 3 = 72$$

$$24 \times 10 = 240$$

$$24 \times 5 = 120$$

### Jottings

\*\*\*\*\*

$$560 \div 24 = 23 \text{ r}8$$

$$\begin{array}{r} 23 \text{ r}8 \\ \hline 24 \overline{) 560} \\ \underline{48} \phantom{0} \\ 80 \\ \underline{72} \\ 8 \end{array}$$



## Long Division

$$432 \div 15$$

$15 \times 1 = 15$   
 $15 \times 2 = 30$   
 $15 \times 3 = 45$   
 $15 \times 10 = 150$   
 $15 \times 5 = 75$

<b>1</b>	<b>5</b>	<b>4</b>	<b>3</b>	<b>2</b>	<b>5</b>   <b>4</b>
		<b>3</b>	<b>0</b>	<b>2</b>	<b>↓</b>
		<b>1</b>	<b>3</b>	<b>2</b>	
		<b>1</b>	<b>2</b>	<b>0</b>	
			<b>1</b>	<b>2</b>	
			<b>1</b>	<b>2</b>	

$\frac{12}{15}$  **Remainder of 12 simplified** – find the greatest common factor that divides into both numerator and the denominator = 3



## Long Division

$$432 \div 15$$

$$\begin{array}{r} 28 \text{ R } 8 \\ 15 \overline{) 432.0} \\ \underline{30} \phantom{0} \\ 132 \phantom{0} \\ \underline{120} \phantom{0} \\ 120 \phantom{0} \\ \underline{120} \\ 0 \end{array}$$

$$(12 \div 15 = 0.8)$$

remainder as a decimal



The Calculation Sequence – applying the skills		
The Sequence	Prompts	Planning
Provide an estimate for the calculation	Using knowledge of number and the number system, rounding and approximating, make a reasonable estimate.	
Teach the calculation skill	What is the objective you are teaching? Include example questions, increasing in complexity, for both operations.	
Ensure you have taught the inverse	Plan example questions, increasing in complexity. Ensure methods used are in line with school calculation policy. Check that children understand that inverse can also be used to check calculations	
Devise similar calculations but include units	Which units do you need to include? Check the measures applicable to your year group for length, weight, capacity, money and time.	
Complete missing box questions	include units in these questions as above. The box may cover single digits or an entire number. Vary the position of the missing box within the calculation.	
Complete word problems, 1 and 2 step, including units	Write problems, ensuring the numbers are sized correctly in line with the objective and that units are also used.	
Provide opportunities for open ended investigations	Plan example questions and investigations. Ensure children are working with the correct operations, appropriate size of numbers and use of units for context.	



THANK YOU